ESERCIZI DI MICROECONOMIA

1) Funzione Marginale Elasticità

$$d = 200 - 2p + \frac{1}{10^3} C^2$$

dove p = prezzo
C = Reddito Consumatore

Calcoliamo prima la derivata rispetto a p e poi a C.

$$\frac{d}{dp} = -2 \qquad \frac{d}{dC} = \frac{1}{500} y$$

Ponendo $p = 10$ e $C = 2000$ avremo

$$d = 200 - 2(10) + \frac{1}{1000} \cdot 2000^2 = 200 - 20 + 4000 =$$

$$= \underline{4180}$$

Quindi avremo:

d rispetto al prezzo $= -2$

d rispetto al C $= \frac{1}{500} \cdot 2000 = 4$

Calcoliamo ora l'Elasticità:

$$\varepsilon_{Lp} = \frac{p}{L} \cdot \frac{dL}{dp} = \frac{p}{200-2p+\frac{1}{10^3}e^2} \cdot (-2) =$$

$$\varepsilon_{Lp} = \frac{-2p}{200-2p+\frac{1}{10^3}e^2} = \frac{-2(10)}{200-2(10)+\frac{1}{10^3}\cdot 2000^2} =$$

$$= \frac{-20}{4180} = -0{,}0047$$

$$\varepsilon_{de} = \frac{C}{dL} \cdot \frac{dL}{de} = \frac{c}{200-2p+\frac{1}{10^3}C^2} \cdot \frac{1}{500}C$$

$$= \frac{c}{200-20+4000} \cdot \frac{1}{500} \cdot 2000 = \frac{2000 \cdot 4}{4180} = \boxed{1,91}$$

Se $\varepsilon > 1$ ELASTICA

Se $\varepsilon = 1$ UNITARIA

Se $\varepsilon < 1$ RIGIDA

Quindi rispetto a p è rigida
Rispetto a C è elastica.

Ex2) $d = 200 - 3p + \sqrt[5]{e}$ $p = 15$ $e = 25.000$

$\frac{dL}{dx} = -3$; $\frac{dL}{dy} = \frac{1}{5y^{\frac{4}{5}}}$.

$d = 200 - 3(15) + \sqrt[5]{25.000} = 200 - 45 + 7,57 = 162,57$

$\varepsilon_{d_p} = \frac{p}{d} \cdot \frac{d}{dp} = \frac{15}{200 - 3p + \sqrt[5]{25.000}} \cdot -3 = \frac{-45}{162,57} = -0,27$

$$\varepsilon_c = \frac{c}{d} \cdot \frac{d}{dc} = \frac{25.000}{900 - 3p + \sqrt[5]{25.000}} \cdot \frac{1}{5c^{\frac{4}{5}}} =$$

$$= \frac{25.000}{162,57} \cdot \frac{1}{5 \cdot (25.000)^{\frac{4}{5}}} = \frac{25.000}{162,57} \cdot \frac{1}{16494} =$$

$$\approx \frac{25.000}{2681430} = \boxed{0,0093}$$

Rigida rispetto al p

Rigida rispetto a c

Ex3) $d = 700 - 14p + \sqrt{e}$ $\quad p = 18 \quad e = 15.000$

$$\frac{d}{dp} = -14 \quad ; \quad \frac{d}{de} = \frac{1}{2\sqrt{e}}$$

$$d = 700 - 14(18) + \sqrt{15000} = 700 - 252 + 122,47 = 570,47$$

$$\varepsilon_{dp} = \frac{p}{d} \cdot \frac{d}{dp} = \frac{18}{700 - 14p + \sqrt{e}} \cdot -14 = \frac{-252}{570,47} = \boxed{-0,441}$$

$$E_{dc} = \frac{P}{d} \cdot \frac{d}{de} = \frac{15}{700-14p+\sqrt{e}} \cdot \frac{1}{2\sqrt{e}} =$$

$$= \frac{15}{700-14(18)+\sqrt{15000}} \cdot \frac{1}{2\sqrt{15000}} =$$

$$= \frac{15}{570,47} \cdot \frac{1}{245} = \frac{15}{139690} = \boxed{0,000107}$$

Es 4) $d = 500 - \frac{1}{2}p + 4\sqrt[3]{e}$; $p = 300$ $e = 20{,}000$

$\frac{\partial d}{\partial p} = -\frac{1}{2}$; $\frac{\partial d}{\partial e} = \frac{4}{3y^{\frac{2}{3}}}$;

$d = 500 - \frac{1}{2}(300) + 4\sqrt[3]{20.000} = 500 - 150 + 108{,}5 = \boxed{458{,}5}$

$\varepsilon_{d_p} = \frac{P}{d} \cdot \frac{d}{d_p} = \frac{300}{458{,}5} \cdot -\frac{1}{2} = \frac{-300}{917} = \boxed{-0{,}327}$

$$\varepsilon_{de} = \frac{c}{d} \cdot \frac{d}{de} = \frac{90.000}{458,5} \cdot \frac{4}{3(90.000)^{2/3}} = \frac{90.000}{458,5} \cdot \frac{4}{2210}.$$

$$= \frac{80.000}{1013985} = 0,078$$

εd_p = RIGIDA

ε_{de} = RIGIDA

FUNZIONE DI DOMANDA DI UN BENE CON PRESENZA DI ALTRO BENE A REDDITO DATO.

$$d = 200 - 4p_x + 0{,}2p_y + 0{,}02C$$

$p_x = 60$ $p_y = 75$ $C = 4000$

$$\frac{d}{dp_x} = -4 \quad ; \quad \frac{d}{dp_y} = 0{,}2 \quad ; \quad \frac{d}{dC} = 0{,}02$$

$$d = 200 - 4(60) + 0{,}2(75) + 0{,}02(4000)$$

$$d = 200 - 240 + 15 + 80 = \boxed{55}$$

$$\varepsilon_{px} = \frac{p_x}{d} \cdot \frac{d}{dx} = \frac{60}{55} \cdot -4 = \frac{-240}{55} = \boxed{-4{,}3}$$

$$\varepsilon_{py} = \frac{75}{55} \cdot 0{,}2 = \boxed{0{,}27}$$

$$\varepsilon_{pe} = \frac{4000}{55} \cdot 0{,}02 = \boxed{1{,}45}$$

Ora Calcoliamo l'Elasticità incrociata tra i due beni. Ipotizziamo un aumento di py del 14%.

$P_y = 75 + 14\% = 85,5$

$$\frac{85,5}{55} \cdot 0,2 = \boxed{0,31}$$

$$|0,2\bar{7} - 0,31| = |0,033|$$

Se py aumenta del 14% la domanda di x aumenta del 3,3%

Ex 5

$$d = 600 - 3p_x + 0{,}5p_y + 0{,}02e$$

$$p_x = 30 \quad p_y = 40 \quad e = 2000$$

$$\frac{d}{dx} = -3 \quad ; \quad \frac{d}{dy} = +0{,}5 \quad ; \quad \frac{d}{de} = 0{,}02$$

$$d = 600 - 3(30) + 0{,}5(40) + 0{,}02(2000) =$$

$$d = 600 - 90 + 20 + 40 = 570$$

$$\varepsilon_{d_x} = \frac{P_x}{d} \cdot \frac{\partial d}{\partial_x} = \frac{30}{570} \cdot -3 = \frac{-90}{570} \boxed{= -0{,}15}$$

$$\varepsilon_{d_y} = \frac{P_y}{d} \cdot \frac{\partial d}{\partial y} = \frac{40}{570} \cdot 0{,}5 = \frac{20}{570} = \boxed{0{,}03}$$

$$\varepsilon_{d_e} = \frac{P_e}{d} \cdot \frac{\partial d}{\partial e} = \frac{2000}{570} \cdot 0{,}02 = \frac{40}{570} = \boxed{0{,}07}$$

Calcolare Elasticità Incrociata:

$$\varepsilon = \frac{P_y}{d} \cdot \frac{d}{d_y} = \boxed{0{,}03}$$

Essendo > 0 i due beni sono

SUCCEDANEI

(Cioè se aumento il prezzo di y aumento il consumo di X)

Se $\varepsilon < 0$ sono complementari

Se $\varepsilon = 0$ Beni senza Relazione

Ex6 $d = 400 - 4p_y - 0{,}8p_y + 0{,}02C$

$p_x = 40$; $p_y = 40$ $C = 1500$

$\frac{d}{dx} = -4$; $\frac{d}{dy} = -0{,}8$; $\frac{d}{dC} = 0{,}02$

$d = 400 - 4(40) - 0{,}8(40) + 0{,}02(1500) =$

$d = 400 - 160 - 32 + 30 = \boxed{238}$

$\varepsilon_{px} = \frac{p_x}{d} \cdot \frac{d}{dx} = \frac{40}{238} \cdot -4 = \frac{-160}{238} = \boxed{-0{,}67}$

$$\varepsilon_{py} = \frac{py}{d} \cdot \frac{d}{dy} = \frac{40}{238} \cdot -0,8 = \frac{-32}{238} = \boxed{-0,13}$$

$$\varepsilon_{c} = \frac{C}{d} \cdot \frac{d}{dc} = \frac{1500}{238} \cdot 0,02 = \frac{30}{238} = \boxed{0,12}$$

Elasticità Incrociata è −0,13 –

Beni Complementari.

Ex6) $d = 100 - 3p_x + 0{,}4p_y + 0{,}01e$

$p_x = 50$; $p_y = 70$; $p_e = 3000$

$\frac{d}{dx} = -3$; $\frac{d}{dy} = +0{,}4$; $\frac{d}{de} = 0{,}01$.

$d = 100 - 3(50) + 0{,}4(70) + 0{,}01(3000)$

$d = 100 - 150 + 28 + 30 = 8$

$\varepsilon_x = \frac{p_x}{d} \cdot \frac{d}{dx} = \frac{50}{8} \cdot -3 = \frac{-150}{8} = \boxed{-18{,}75}$

$$\varepsilon_{px} = \frac{p_y}{d} \cdot \frac{d}{d_y} = \frac{70}{8} \cdot 0{,}4 = \frac{28}{8} = \boxed{3{,}5}$$

$$\varepsilon_c = \frac{C}{d} \cdot \frac{d}{d_C} = \frac{3000}{8} \cdot 0{,}01 = \frac{30}{8} = \boxed{3{,}75}$$

Elasticità incrociata $= 3{,}5$.

Beni Succedanei.

Se il prezzo di y aumenta del 10%, come varia la domanda.

$y = 70 + 10\% = 70 + 7 = 77$

$d^2 = 100 - 3(50) + 0{,}4(77) + 0{,}01(3000) =$

$d_2 = 100 - 150 + 30{,}8 + 30 = \boxed{10{,}8}$

La nuova domanda (d_2) passa da 8 a 10,8

Aumento del $= 10{,}8 - 8 = \frac{2{,}8 \times 100}{8} = \boxed{35\%}$

7) $d = 200 - 6px - 2py + 0{,}5C$

$px = 35$; $py = 35$. $C = 3500$

$\frac{d}{dx} = -6$; $\frac{d}{dy} = -2$; $\frac{d}{dC} = 0{,}5$

$d = 200 - 6(35) - 2(35) + 0{,}5(3.500) =$

$d = 200 - 210 - 70 + 1750 = \boxed{1670}$

$$\varepsilon_{px} = \frac{p_x}{d} \cdot \frac{\partial d}{\partial x} = \frac{35}{1670} \cdot -6 = \frac{-210}{1670} = \boxed{-0,12}$$

$$\varepsilon_{py} = \frac{p_y}{d} \cdot \frac{\partial d}{\partial y} = \frac{35}{1670} \cdot -2 = \frac{-70}{1670} = \boxed{-0,041}$$

$$\varepsilon_{e} = \frac{e}{d} \cdot \frac{\partial d}{\partial e} = \frac{3500}{1670} \cdot 0,5 = \frac{1750}{1670} = \boxed{1,047}$$

Il prezzo di y aumenta dell'8%.

$$35 + 8\% = 35 + 2{,}8 = \boxed{37{,}8}$$

$$d_2 = 200 - 6(35) - 2(37{,}8) + 0{,}5(3500)$$

$$d_2 = 200 - 210 - 75{,}6 + 1750 = 1664{,}4$$

La differenza è di $1670 - 1664{,}4 = \boxed{5{,}6}$

cioè $\dfrac{5{,}6 \times 100}{1664{,}4} \times 100 = \boxed{-0{,}33\%}$

8) $d = -0{,}8\,p_x^2 + 0{,}9\,p_y^2 + 0{,}3\,e$

$p_x = 60$; $p_y = 80$; $e = 5000$

$\frac{d}{dx} = -1{,}6x$; $\frac{d}{dy} = 1{,}8y$; $\frac{d}{de} = 0{,}3$

$d = -0{,}8(60)^2 + 0{,}9(80)^2 + 0{,}3\,(5000)$

$d = -2880 + 5760 + 1500 = \boxed{4380}$

$$\varepsilon_{px} = \frac{\partial x}{\partial} \cdot \frac{\partial}{\partial x} = \frac{60}{4380} \cdot -1,6(60) = -\frac{5760}{4380} = \boxed{-1,31}$$

$$\varepsilon_{py} = \frac{py}{\partial} \cdot \frac{\partial}{\partial y} = \frac{80}{4380} \cdot 1,8(80) = \frac{11520}{4380} = \boxed{2,63}$$

$$\varepsilon_{c} = \frac{c}{\partial} \cdot \frac{\partial}{\partial c} = \frac{5000}{4380} \cdot 0,3 = \boxed{0,34}$$

Beni succedanei.

8) $d = 800 - 0{,}02 p_x^2 + \sqrt{c}$

$p_x = 100$; $c = 16.000$

$\sqrt{}$

16×1000

$4^2 \times 10^3$

4^2

$\frac{d}{dx} = -0{,}04 p_x$; $\frac{d}{dc} = \frac{1}{2\sqrt{c}}$

$d = 800 - 0{,}02(100)^2 + \sqrt{16.000} =$

$d = 800 - 200 + 126{,}50 = 726{,}5$

$$\varepsilon_{dx} = \frac{P_x}{d} \cdot \frac{d}{dx} = \frac{100}{726,5} \cdot -0{,}04(100) = \frac{-400}{726,5}$$

$$\varepsilon_{e} = \frac{e}{d} \cdot \frac{d}{de} = \frac{16.000}{726,5} \cdot \frac{1}{2\sqrt{16.000}} = \frac{16.000}{726,5} \cdot \frac{1}{2 \cdot 126{,}50} =$$

$$= \frac{16.000}{726,5} \cdot \frac{1}{252,8} = \frac{16.000}{183731,8} =$$

g) $d = 6000 - p_x^2 + 0{,}3 p_y + 0{,}06 e$

$p_x = 80$; $p_y = 70$; $e = 12.000$

$\frac{d}{dx} = 2p_x$; $\frac{d}{dy} = 0{,}3$; $\frac{d}{de} = 0{,}06$

$d = 6000 - 80^2 + 0{,}3(70) + 0{,}06(12.000)$

$d = 6000 - 6400 + 21 + 720$

$d = 341$

$$\varepsilon_{px} = \frac{px}{d} \cdot \frac{d}{dx} = \frac{80}{341} \cdot 2(80) = \frac{12800}{341} = \boxed{-37,53}$$

$$\varepsilon_{py} = \frac{py}{d} \cdot \frac{d}{dy} = \frac{70}{341} \cdot 0,3 = \boxed{0,061}$$

$$\varepsilon_{c} = \frac{c}{d} \cdot \frac{d}{dc} = \frac{12.000}{341} \cdot 0,06 = \boxed{2,11}$$

Beni Succedanei.

Ex 10) SCELTA OTTIMA CONSUMATORE

$$\text{Utilità} = (x+5)^2 \cdot (y+2)^2$$

$x = 16$ unità

$y = 10$ unità

$$\frac{dL}{dx} = 2(x+5)(y+2)^2 = 2(16+5)(10+2)^2 = 42 \cdot 144 = \boxed{6048}$$

$$\frac{dL}{dy} = 2(x+5)^2(y+2) = 2(16+5)^2(10+2) = 882 \cdot 12 = \boxed{10584}$$

Ex 11 $U = x^2 y$ $x = 12$ $y = 3$

$$\frac{dU}{dx} = 2xy \quad ; \quad \frac{dU}{dy} = x^2$$

$$U_x = 2(12)(3) = \boxed{216}$$

$$U_y = (12)^2 = \boxed{144}$$

Ex 12) $U = 100 \ln(x+5) + y^2$

$x = 5 \; ; \; y = 10$

$\frac{d}{dx} = \frac{100}{x+5} \; ; \; \frac{d}{dy} = 2y$

$U_x = \frac{100}{5+5} = \frac{100}{10} = \boxed{10}$

$U_y = 2(10) = \boxed{20}$

Ex 13 $\quad U = \sqrt{xy} \qquad x = 4 \; ; \; y = 25$

$$\frac{dU}{dx} = \frac{y}{2\sqrt{xy}} \; ; \quad \frac{dU}{dy} = \frac{x}{2\sqrt{xy}} \; ;$$

$$U_x = \frac{25}{2\sqrt{4 \cdot 25}} = \frac{25}{2\sqrt{100}} = \frac{25}{2 \cdot 10} = \frac{25}{20} = \boxed{1{,}25}$$

$$U_y = \frac{4}{2\sqrt{100}} = \frac{4}{20} = \boxed{0{,}2}$$

Ex 14 $U = 100 \ln(x+5) + 10 \ln y^2$

$x = 15 \;;\; y = 20$

$$\frac{d}{dx} = \frac{100}{x+5} \;;\quad \frac{d}{dy} = \frac{20 \log(y)}{y}.$$

$$U_x = \frac{100}{15+5} = \frac{100}{20} = \boxed{5}.$$

$$U_y = \frac{20}{20} = \boxed{1}$$

Ex15) $U = xy + x + y + 2$.

$p_x = 4$; $p_y = 6$ Reddito $= 110$

$\frac{d}{dx} = y+1$; $\frac{d}{dy} = x+1$.

$$\begin{cases} \frac{y+1}{x+1} = \frac{4}{6} \\ 4x + 6y = 110 \end{cases} \rightarrow \begin{cases} \frac{6(y+1)}{\cancel{6(x+1)}} = \frac{4(x+1)}{\cancel{6(x+1)}} \\ 4x + 6y = 110 \end{cases} \rightarrow \begin{cases} 6y + 6 = 4x + 4 \\ 4x + 6y = 110 \end{cases}$$

$\searrow$

$$\begin{cases} 6y = 4x + 4 - 6 \\ 4x + 6y = 110 \end{cases} \rightarrow \begin{cases} y = \dfrac{4x-2}{6} \\ 4x + 6\left(\dfrac{4x-2}{6}\right) = 110 \end{cases} \rightarrow \begin{cases} y = \dfrac{4x-2}{6} \\ 4x + 4x - 2 = 110 \end{cases}$$

$$\begin{cases} y = \dfrac{4x-2}{6} \\ 8x = 112 \end{cases} \rightarrow \begin{cases} y = \dfrac{4x-2}{6} \\ x = \dfrac{112}{8} = \boxed{14} \end{cases} \rightarrow y = \dfrac{4(14) - 2}{6} =$$

$$y = \frac{54}{6} = \boxed{9}$$

Scelta Ottima $(14 ; 9)$

Ex16 $U = (x+2)\cdot(y+3)$ Reddito = 97

$p_x = 4$; $p_y = 5$

$\frac{dU}{dx} = y+3$; $\frac{dU}{dy} = 2+x$

$$\begin{cases} \frac{y+3}{2+x} = \frac{4}{5} \\ 4x+5y=97 \end{cases} \rightarrow \begin{cases} \frac{5(y+3)}{5(2+x)} = \frac{4(2+x)}{5(2+x)} \\ 4x+5y=97 \end{cases} \rightarrow \begin{cases} 5y+15=8+4x \\ 4x+5y=97 \end{cases}$$

$$\begin{cases} 5y = 8 + 4x - 15 \\ 4x + 5y = 97 \end{cases} \rightarrow \begin{cases} y = \frac{4x-7}{5} \\ 4x + 5\left(\frac{4x-7}{5}\right) = 97 \end{cases} \rightarrow \begin{cases} y = \frac{4x-7}{5} \\ 4x + 4x - 7 = 97 \end{cases}$$

$$\begin{cases} y = \frac{4x-7}{5} \\ 8x = 104 \end{cases} \rightarrow \begin{cases} y = \frac{4x-7}{5} \\ x = \frac{104}{8} = \boxed{13} \end{cases} \rightarrow y = \frac{4(13)-7}{5} = \quad y = \boxed{9}$$

Scelta ottima (13 ; 9)

Es. 17) $U = xy$; $R = 100$; $p_x = 10$; $p_y = 20$

$\frac{d}{dx} = y$; $\frac{d}{dy} = x$;

$$\begin{cases} \frac{y}{x} = \frac{10}{20} \\ 10x + 20y = 100 \end{cases} \rightarrow \begin{cases} \frac{20y}{20x} = \frac{10x}{20x} \\ 10x + 20y = 100 \end{cases}$$

$$\begin{cases} y = \frac{10}{20}x \\ 10x + 20^{10}\left(\frac{1}{2_1}x\right) = 100 \end{cases} \rightarrow \begin{cases} y = \frac{1}{2}x \\ 10x + 10x = 100 \end{cases} \rightarrow \begin{cases} y = \frac{1}{2}x \\ 20x = 100 \end{cases}$$

$\searrow$

$$\begin{cases} y = \frac{1}{2}x \\ x = \frac{100}{20} = \boxed{5} \end{cases} \rightarrow y = \frac{1}{2}(5) = \boxed{\frac{5}{2}}$$

Scelta ottima $\left(5; \frac{5}{2}\right)$

Es. 18 $U = 10^{-3}(x + yx + y)$ $R = 3000$

$p_x = 20$; $p_y = 30$

$$\frac{dL}{dx} = \left(\frac{1+y}{1000}\right) ; \quad \frac{dL}{dy} = \frac{(1+x)}{1000} ;$$

$$\begin{cases} \frac{1+y}{1000} \cdot \frac{1000}{1+x} = \frac{20}{30} \\ 20x + 30y = 3000 \end{cases} \longrightarrow \begin{cases} \frac{1+y}{1+x} = \frac{2\not{0}}{3\not{0}} \\ 20x + 30y = 3000 \end{cases}$$

$\searrow$

$$\begin{cases} \dfrac{3(1+y)}{\cancel{3(1+x)}} = \dfrac{2(1+x)}{\cancel{3(1+x)}} \\ 20x+30y=3000 \end{cases} \longrightarrow \begin{cases} 3+3y=2+2x \\ 20x+30y=3000 \end{cases}$$

$$\begin{cases} 3y=2x-1 \\ 20x+30y=3000 \end{cases} \longrightarrow \begin{cases} y=\dfrac{2x-1}{3} \\ 20x+\overset{10}{\cancel{30}}\left(\dfrac{2x-1}{\cancel{3}}\right)=3000 \end{cases}$$

$$\searrow$$

$$\begin{cases} y = \dfrac{2x-1}{3} \\ 20x + 20x - 10 = 3000 \end{cases} \rightarrow \begin{cases} y = \dfrac{2x-1}{3} \\ 40x = 3010 \end{cases} \begin{cases} y = \dfrac{2x-1}{3} \\ x = \dfrac{3010}{40} = \boxed{75,2} \end{cases}$$

$$y = \frac{2(75,2)-1}{3} = \boxed{49,8}$$

Scelta ottima $(75,2 ; 49,8)$

Es. 13 $U = 10 + xy$; $R = 240$; $p_x = 20$; $p_y = 40$

$\frac{d}{dx} = y$; $\frac{d}{dy} = x$

$\begin{cases} \frac{y}{x} = \frac{20}{40} \\ 20x + 40y = 240 \end{cases} \rightarrow \begin{cases} \frac{4y}{4x} = \frac{2x}{4x} \\ 20x + 40y = 240 \end{cases} \rightarrow \begin{cases} y = \frac{2x}{4} \\ 20x + 40\left(\frac{2}{4}x\right) = 240 \end{cases}$

$\begin{cases} y = \frac{2}{4}x \\ 20x + 20x = 240 \end{cases} \rightarrow \begin{cases} y = \frac{2}{4}x \\ 40x = 240 \end{cases} \rightarrow \begin{cases} y = \frac{2}{4}x \\ x = \frac{240}{40} = \boxed{6} \end{cases} \rightarrow y = \frac{2}{4}(6) = \boxed{3}$

Scelta ottima $(6; 3)$

Es. 20 $U = \frac{xy}{1000}$; $R = 5000$; $p_x = 2$; $p_y = 1$

$\frac{dU}{dx} = \frac{y}{1000}$; $\frac{dU}{dy} = \frac{x}{1000}$:

$$\begin{cases} \frac{y}{1000} \cdot \frac{1000}{x} = \frac{2}{1} \\ 2x + 1y = 5000 \end{cases} \rightarrow \begin{cases} \frac{y}{x} = \frac{2}{1} \\ 2x + 1y = 5000 \end{cases}$$

$$\begin{cases} y = 2x \\ 2x + 2x = 5000 \end{cases} \quad \begin{cases} y = 2x \\ 4x = 5000 \end{cases} \quad \begin{cases} y = 2x \\ x = \frac{5000}{4} = \boxed{1250} \end{cases}$$

$$y = 2 \cdot (1250) = \boxed{2500}$$

Scelta ottima : (1250 ; 2500)

Es. 21 $U = x + xy$ $\qquad x + 5y = 60$

$\frac{d}{dx} = 1+y \; ; \; \frac{d}{dy} = x$

$\begin{cases} \frac{1+y}{x} = \frac{1}{5} \\ x + 5y = 60 \end{cases}$ $\begin{cases} \frac{5(1+y)}{5x} = \frac{x}{5x} \\ x+5y=60 \end{cases}$ $\begin{cases} 5+5y = x \\ x+5y=60 \end{cases}$ $\begin{cases} y = \frac{x-5}{5} \\ x+5\left(\frac{x-5}{5}\right)=60 \end{cases}$

$\begin{cases} y = \frac{x-5}{5} \\ x+x-5=60 \end{cases}$ $\begin{cases} y = \frac{x-5}{5} \\ 2x=65 \end{cases}$ $\begin{cases} y = \frac{x-5}{5} = \\ x = \frac{65}{2} = \boxed{32,5} \end{cases}$ ✗

$$\begin{cases} y = \dfrac{x-5}{5} = \dfrac{32{,}5-5}{5} = \boxed{5{,}5} \\ \text{Scelta Ottimo } (32{,}5 \, ; \, 5{,}5) \end{cases}$$

Es. 22 $U = -x^2 - 16y$; $R = 500$

$p_x = 2$; $p_y = 4$

$\frac{dL}{dx} = -2x$; $\frac{dL}{dy} = -16$

$$\begin{cases} \frac{-2x}{-16} = \frac{2}{4} \\ 2x + 4y = 500 \end{cases} \rightarrow \begin{cases} \frac{2x}{\cancel{16}} = \frac{8}{\cancel{16}} \\ 2x + 4y = 500 \end{cases} \rightarrow \begin{cases} x = \frac{\cancel{8}}{\cancel{2}} = 4 \\ 2(4) + 4y = 500 \end{cases}$$

$\searrow$

$$\begin{cases} x=4 \\ 8+4y=500 \end{cases} \longrightarrow \begin{cases} x=4 \\ y=\frac{500-8}{4}=123 \end{cases}$$

Ottimo del Consumatore $(4;123)$

Es. 23

$$U = 100 - \frac{1}{x} - \frac{1}{y} \; ; \quad R = 3600$$

$$P_x = 8 \; ; \quad P_y = 4$$

$$\frac{dU}{dx} = \frac{1}{x^2} \; ; \quad \frac{dU}{dy} = \frac{1}{y^2} .$$

$$\begin{cases} \frac{1}{x^2} \cdot y^2 = \frac{8}{4} \\ 8x + 4y = 3600 \end{cases}$$

$$\begin{cases} \frac{4y^2}{\cancel{4x^2}} = \frac{8x^2}{\cancel{4x^2}} \\ 8x + 4y = 3600 \end{cases} \longrightarrow \begin{cases} y^2 = \frac{8x^2}{4} \\ 8x + 4y = 3600 \end{cases} \longrightarrow \begin{cases} y = \sqrt{\frac{8x^2}{4}} \\ 8x + 4y = 3600 \end{cases}$$

$$\begin{cases} y = \frac{3}{2}x \\ 9x + \overset{2}{\cancel{4}}\left(\frac{3}{\cancel{2}_1}x\right) = 3600 \end{cases} \qquad \begin{cases} y = \frac{3}{2}x \\ 9x + 6x = 3600 \end{cases}$$

$$\begin{cases} y = \frac{3}{2}x \\ 15x = 3600 \end{cases} \qquad \begin{cases} y = \frac{3}{2}x \\ x = \frac{3600}{15} = \boxed{240} \end{cases} \qquad \begin{cases} y = \frac{3}{2}(240) = \boxed{360} \\ \end{cases}$$

Scelta Ottima $(240 ; 360)$

Es.24 $U = \sqrt[3]{x^2} + \sqrt[3]{y}$; $R = 11.250$

$P_x = 30$; $P_y = 15$

$\frac{d}{dx} = \frac{2}{\sqrt[3]{3x}}$; $\frac{d}{dy} = \frac{1}{\sqrt[3]{3y^2}}$

$$\begin{cases} \frac{2}{\sqrt[3]{3x}} \cdot \sqrt[3]{3y^2} = \frac{30}{15} \\ 30x + 15y = 11.250 \end{cases} \longrightarrow \begin{cases} \frac{2y^2}{x} = 2 \\ 30x + 15y = 11.250 \end{cases}$$

↘

$$\begin{cases} \dfrac{2y^2}{\cancel{x}} = \dfrac{2x}{\cancel{x}} \\ 30x + 15y = 11.250 \end{cases} \longrightarrow \begin{cases} y^2 = \dfrac{\cancel{2}x}{\cancel{2}} \\ 30x + 15y = 11.250 \end{cases} \longrightarrow \begin{cases} x = y^2 \\ 30(y^2) + 15y = 11250 \end{cases}$$

$$\begin{cases} x = y^2 \\ y_{1|2} = \dfrac{-15 \pm \sqrt{225 + 1350000}}{60} \end{cases} = \begin{cases} \dfrac{-15 + 1162}{60} = \boxed{19,11} \\ \dfrac{-15 - 1162}{60} = -19,6 \text{ non valida} \end{cases}$$

$$x = (19,1)^2 = \boxed{364}$$

$$(364 ; 19)$$

Es 25 $U = x^2 y$; $R = 1500$; $p_x = 1$; $p_y = 5$

$\frac{dL}{dx} = 2xy$; $\frac{dL}{dy} = x^2$

$$\begin{cases} \frac{2\cancel{x}y}{x^{\cancel{2}}} = \frac{1}{5} \\ 1x + 5y = 1500 \end{cases} \rightarrow \begin{cases} \frac{5(2y)}{\cancel{5x}} = \frac{x}{\cancel{5x}} \\ 1x + 5y = 1500 \end{cases} \rightarrow \begin{cases} 10y = x \\ 1x + 5y = 1500 \end{cases}$$

$$\begin{cases} y = \frac{x}{10} \\ 1x + \cancel{5}\left(\frac{x}{\cancel{10}\,2}\right) = 1500 \end{cases} \rightarrow \begin{cases} y = \frac{x}{10} \\ 2x + x = 3000 \end{cases} \rightarrow

$$\begin{cases} y = \dfrac{x}{10} \\ 3x = 3000 \end{cases} \longrightarrow \begin{cases} y = \dfrac{x}{10} \\ x = \dfrac{3000}{3} = \boxed{1000} \end{cases}$$

$$y = \frac{1000}{10} = \boxed{10}$$

Scelto Ottimo $(1000; 100)$

Es. 26 Il Problema del Produttore

Un'impresa per produrre un bene utilizza due fattori produttivi, x e y, e presenta il seguente vincolo $g(x;y) = 4x+16y-36$

con $Q(x;y) = 2x^2y$.

Determinare la quantità massima che è possibile produrre.

$$\frac{d}{dx} = 4xy \quad ; \quad \frac{d}{dy} = 2x^2$$

$$\begin{cases} \dfrac{4xy}{2x^2} = \dfrac{4}{16} \\ 4x + 16y = 96 \end{cases} \rightarrow \begin{cases} \dfrac{2y}{x} = \dfrac{1}{4} \\ 4x+16y=96 \end{cases} \rightarrow \begin{cases} \dfrac{8y}{4x} = \dfrac{x}{4x} \\ 4x+16y=96 \end{cases}$$

$$\begin{cases} y = \dfrac{1}{8}x \\ 4x + 16\left(\dfrac{1}{8}x\right) = 96 \end{cases} \rightarrow \begin{cases} y = \dfrac{1}{8}x \\ 4x+2x=96 \end{cases} \rightarrow \begin{cases} y = \dfrac{1}{8}x \\ 6x = 96 \end{cases} \rightarrow \begin{cases} y = \dfrac{1}{8}x \\ x = \dfrac{96}{6} = \boxed{16} \end{cases}$$

$$y = \frac{1}{8} \cdot 16 = \boxed{2} \qquad \text{MAX } (16 ; 2)$$

Es. 27 $\quad g(x;y) = 20x + 80y - 4000 \quad ; \quad Q = 10\sqrt{xy}$

$$\frac{dL}{dx} = \frac{5y}{\sqrt{xy}} \quad ; \quad \frac{dL}{dy} = \frac{5x}{\sqrt{xy}}$$

$$\begin{cases} \dfrac{5y}{\sqrt{xy}} \cdot \dfrac{\sqrt{xy}}{5x} = \dfrac{120}{480} \\ 20x + 80y = 4000 \end{cases}$$

$$\begin{cases} \dfrac{y}{x} = \dfrac{1}{4} \\ 20x + 80y = 4000 \end{cases} \longrightarrow \begin{cases} \dfrac{4y = x}{4x} \\ 20x + 80y = 4000 \end{cases} \longrightarrow \begin{cases} y = \dfrac{1}{4}x \\ 20x + \overset{20}{80}\left(\dfrac{1}{4}x\right) = 4000 \end{cases}$$

$$40x = 4000 \qquad x = \frac{4000}{40} = \boxed{100} \qquad y = \frac{1}{4}(100) = \boxed{25}$$

Es. 28) Produzione in Concorrenza Perfetta

Un'impresa produce due beni A e B in concorrenza perfetta al prezzo di 3 per A e di 2 per B.
La funzione del Costo è:

$$C(x;y) = 40 + 0{,}05x + 0{,}1y + 0{,}0002x^2 + 0{,}001y^2$$

Determiniamo le quantità che rendono il profitto massimo e l'ammontare dello stesso.

$$\frac{dL}{dx} = \frac{1}{20} + \frac{x}{25000} \quad ; \quad \frac{dL}{dy} = \frac{1}{10} + \frac{y}{500}$$

In regime di concorrenza perfetta il Prezzo deve essere uguale al Costo Marginale. $P = CMg$

Quindi

$$3 = \frac{1}{20} + \frac{x}{25.000} = \frac{75.000 = 1250 + x}{25.000} =$$

$$x = 75.000 - 1250 = \boxed{73750}$$

$$y = \frac{1}{10} + \frac{y}{500} = 2 = \frac{1}{10} + \frac{y}{500} = \frac{1000 = 50 + y}{\cancel{500}}$$

$$y = 1000 - 50 = \boxed{950}$$

Le quantità sono $x = 73.750$

$y = 950$

Il Profitto si trova con:

$$C = 40 + 0,05(73.750) + 0,1(950) + 0,000002(73.750)^2 + 0,001(950)^2 =$$

$$C = 40 + 3687,5 + 95 + 10878 1,25 + 902,5 =$$

$$C = 113506,25$$

$$\text{Ricavo} = P \times Q = (3 \times 73.750) + (2 \times 950) =$$

$R = 221.250 + 1.900 = 223.150$

Profitto $(\pi) = 223.150 - 113.506,25 = \boxed{109.643,75}$